Chiral symmetry breaking from five dimensional spaces

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Abstract

Based on the AdS/CFT correspondence we study the breaking of the chiral symmetry in QCD using a simple five dimensional model. The model gives definite predictions for the spectrum of vector mesons, their decay constants and interactions as a function of one parameter related to the quark condensate. We calculate the coefficients L_i of the low-energy QCD chiral lagrangian, as well as other physical quantities for the pions. All the predictions are shown to be in good agreement with the experimental data. We also show that they are robust under modifications of the 5D metric in the IR, and that some of them arise as a consequence of the higher-dimensional gauge symmetry. For example, at the tree-level, we find $M_{\rho} \simeq \sqrt{3} \, g_{\rho\pi\pi} F_{\pi}$, $F_{\rho} \simeq \sqrt{3} \, F_{\pi}$ and $\mathrm{BR}(a_1 \to \pi \gamma) = 0$.

1 Introduction

The string/gauge duality [1] has allowed us in the last years to gain new insights into the problem of strongly coupled gauge theories. Although a string description of real QCD has not yet been formulated, different string constructions have been able to describe gauge theories with certain similarities to QCD. Recently, the incorporation of D7-branes in the $AdS_5 \times S^5$ background [2] has allowed to address flavor issues [3].

A related but more phenomenological approach to QCD has consisted in extracting properties of QCD using 5D field theories in Anti-de-Sitter (AdS) [4, 5, 6, 7, 8]. This approach is based on the AdS/CFT correspondence [9] that relates strongly coupled conformal field theories (CFT) to weakly coupled 5D theories in AdS. This is a more modest attempt but, in certain regimes, it grasps the generic features of the more involved string constructions.

This approach can be useful to study chiral symmetry breaking in the vector and axial-vector sector of QCD. It is known from the OPE that the vector-vector current correlator for large Euclidean momentum, $p \gg \Lambda_{\rm QCD}$, is given in the chiral limit by [10]

$$\Pi_V(p^2) = p^2 \left[\beta \ln \frac{\mu^2}{p^2} + \frac{\gamma}{p^4} + \frac{\delta}{p^6} + \cdots \right],$$
(1)

where $\beta \simeq N_c/(12\pi^2)$, $\gamma \simeq \alpha_s \langle G_{\mu\nu}^2 \rangle / 12\pi$ and $\delta \simeq -28\pi\alpha_s \langle \bar{q}q \rangle^2 / 9$ are almost momentum-independent coefficients. Similar expression holds for the axial-axial correlator Π_A . Therefore QCD behaves in Eq. (1) as a near-conformal theory in the ultraviolet (UV) in which the breaking of the conformal symmetry is given by the condensates. The correlator Π_V , on the other hand, must have, according to the large- N_c expansion, single poles in the imaginary axis of p corresponding to colorless vector resonances. These properties of QCD can be implemented in a 5D theory in AdS. The condensates $\langle \mathcal{O} \rangle$ are described, in the AdS side, by vacuum expectation values (VEV) of scalars Φ whose masses are related to the dimension d of \mathcal{O} by [9] $d = \sqrt{4 + M_{\Phi}^2 L^2} + 2$ (L is the AdS curvature radius), while confinement and the mass gap in QCD can be obtained in the AdS₅ by compactifying the fifth dimension. Alike large- N_c QCD, the 5D theory is also described as a function of weakly coupled states corresponding to the mesons.

In this article we will present a simple 5D model to study chiral symmetry breaking in QCD. We will calculate the vector and axial correlators, Π_V and Π_A , and derive from them the masses and decay constants of the vector, axial-vector and pseudo-Goldstone (PGB) mesons. We will also calculate their interactions and show some generic properties of 5D models. As an example, we will study the electromagnetic form factor of the pions and show how vector-meson dominance (VMD) appears. Finally, we will derive the PGB chiral lagrangian arising from this 5D model and we will give the predictions for the L_i coefficients as well as for the PGB masses. We will compare all these predictions with the experimental data.

The model presented here can also be useful to study electroweak symmetry breaking along the lines of Refs. [11, 12, 13, 14]. For this reason in the appendix we study the generic case in which the breaking of the chiral or electroweak symmetry arises from an operator of dimension d. This allows us to calculate the dependence on d of the Peskin-Takeuchi [15] S parameter.

2 A 5D model for chiral symmetry breaking

The 5D analog of QCD with 3 flavors consists in a theory with a $SU(3)_L \otimes SU(3)_R$ gauge symmetry in the 5D bulk and a parity defined as the interchange $L \leftrightarrow R$. We will not consider the extra $U(1)_{L,R}$ that involves the anomaly. The 5D metric is defined generically as

$$ds^{2} = a^{2}(z) \left(\eta_{\mu\nu} dx^{\mu} dx^{\nu} - dz^{2} \right), \tag{2}$$

where a is the warp factor that in the case of AdS_5 is given by

$$a(z) = \frac{L}{z},\tag{3}$$

where L is the AdS curvature radius. We will compactify this space by putting two boundaries, one at $z = L_0$ (UV-boundary) and another at $z = L_1$ (IR-boundary). The theory is then defined on the line segment $L_0 \le z \le L_1$ [16]. The UV-boundary acts as a regulator necessary to obtain finite calculations. The limit $L_0 \to 0$ should be taken after divergencies are canceled by adding counterterms on the UV boundary [9]. The IR-boundary is needed to introduce a mass gap in the theory $\sim 1/L_1$.

The only fields in the bulk that we will consider are the gauge boson fields, L_M and R_M , and a scalar field Φ transforming as a $(\mathbf{3_L}, \mathbf{3_R})$ whose VEV will be responsible for the breaking of the chiral symmetry. The action is given by

$$S_5 = \int d^4x \int dz \, \mathcal{L}_5 \,, \tag{4}$$

where

$$\mathcal{L}_5 = \sqrt{g} M_5 \operatorname{Tr} \left[-\frac{1}{4} L_{MN} L^{MN} - \frac{1}{4} R_{MN} R^{MN} + \frac{1}{2} |D_M \Phi|^2 - \frac{1}{2} M_{\Phi}^2 |\Phi|^2 \right] , \tag{5}$$

the covariant derivative is defined as

$$D_M \Phi = \partial_M \Phi + i L_M \Phi - i \Phi R_M \,, \tag{6}$$

and g is the determinant of the metric. We have defined $L_M = L_M^a T^a$ where $M = (\mu, 5)$ and $\text{Tr}[T^a T^b] = \delta_{ab}$, and similarly for the other fields. A coefficient M_5 has been factored out in front of the lagrangian so that $1/\sqrt{M_5}$ is the 5D expansion parameter playing the role of $1/\sqrt{N_c}$ in QCD. We define $\Phi = S e^{iP/v(z)}$ where $v(z) \equiv \langle S \rangle$ and S corresponds to a real scalar and P to a real pseudoscalar $(S \to S \text{ and } P \to -P \text{ under } L \leftrightarrow R)$. They transform as $\mathbf{1} + \mathbf{8}$ under $\mathrm{SU}(3)_V$.

Let us study v(z) in the case of AdS₅. We assume $M_{\Phi}^2 = -3/L^2$ that corresponds in the CFT to an operator of dimension 3 such as $\bar{q}q$. Solving the bulk equation of motion for S we get

$$v(z) = c_1 z + c_2 z^3, (7)$$

where c_1 and c_2 are two integration constants. They can be determined as a function of the value of v(z) at the boundaries:

$$c_1 = \frac{M_q L_1^3 - \xi L_0^2}{L L_1 (L_1^2 - L_0^2)}, \qquad c_2 = \frac{\xi - M_q L_1}{L L_1 (L_1^2 - L_0^2)}, \tag{8}$$

where we have defined

$$M_q \equiv \frac{L}{L_0} v \big|_{L_0} , \qquad \xi \equiv L v \big|_{L_1} . \tag{9}$$

By the AdS/CFT correspondence, a nonzero M_q is equivalent to put an explicit breaking of the chiral symmetry in the CFT (such as adding quark masses). On the other hand, a nonzero value of $\xi \propto 1$ corresponds in the chiral limit, $M_q = 0$, to an spontaneously breaking SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V, playing the role of the condensate $\langle \bar{q}q \rangle$ in QCD. By substituting the solution Eq. (7) back into the action, we obtain the vacuum energy. Taking the limit $L_0 \rightarrow 0$ while keeping M_q fixed, this is given by (up to divergent terms)

$$S_4 \simeq -M_5 L \int d^4 x \operatorname{Tr} \left[\frac{M_q^2}{L_1^2} - 2 \frac{\xi M_q}{L_1^3} + \frac{3}{2} \frac{\xi^2}{L_1^4} + V(\xi) \frac{1}{L_1^4} \right],$$
 (10)

where we have added a potential on the IR-boundary $V(\xi)$. This potential is assumed to exist in order to have a nonzero ξ at the minimum of Eq. (10) even in the chiral limit $M_q = 0$. Possible origins of a potential for ξ are given in Refs. [3]. In the following we will take $\xi \to \xi \mathbb{1}$ where ξ will be considered an input parameter. Therefore the vector sector of the model has 4 parameters, M_5 , M_q , L_1 , and ξ . As we will see M_5 is related to N_c , M_q to the quark masses and $1/L_1$ corresponds to the mass gap to be related to $\Lambda_{\rm QCD}$. The model then has, with respect to QCD, only one extra parameter, ξ .

Few comments are in order. Using naive dimensional analysis one can estimate that this 5D theory becomes strongly coupled at a scale $\sim 24\pi^3 M_5$. This implies that extra (stringy) physics must appear at this scale or, equivalently, that this is the scale that suppresses higher dimensional operators in Eq. (5). We estimate this scale to be around few GeV. Second, we are neglecting the backreaction on the metric due to the presence of the scalar VEV. Although a nonzero energy-momentum tensor of Φ will affect the geometry of the space producing a departure from AdS, this effect will only be relevant at z very close to the IR-boundary, and therefore it will not substantially change our results. We will comment on this in the last section. Notice that neglecting the backreaction corresponds to freeze other possible condensates that turn on in the presence of the quark condensate.

3 Vector, axial-vector and PGB sectors

We are interested in studying the vector, axial-vector and PGB sectors. The scalar sector is more model-dependent and will be left for the future. Let us first consider the chiral limit $M_q = 0$. We take this limit in the following way. First we consider $c_1 \to 0$ with fixed L_0 and,

after performing the calculations, we take $L_0 \to 0$. In the chiral limit we have $v \propto 1$ and then it is convenient to define the vector and axial gauge bosons:

$$V_{M} = \frac{1}{\sqrt{2}} (L_{M} + R_{M}),$$

$$A_{M} = \frac{1}{\sqrt{2}} (L_{M} - R_{M}).$$
(11)

By adding the gauge fixing terms

$$\mathcal{L}_{GF}^{V} = -\frac{M_5 a}{2\xi_V} \operatorname{Tr} \left[\partial_{\mu} V_{\mu} - \frac{\xi_V}{a} \partial_5 (aV_5) \right]^2,
\mathcal{L}_{GF}^{A} = -\frac{M_5 a}{2\xi_A} \operatorname{Tr} \left[\partial_{\mu} A_{\mu} - \frac{\xi_A}{a} \partial_5 (aA_5) - \xi_A \sqrt{2} a^2 v P \right]^2,$$
(12)

the gauge bosons V_{μ} and A_{μ} do not mix with the scalars A_5 and P. We will take the limit $\xi_{V,A} \to \infty$, i.e.

$$P = -\frac{1}{\sqrt{2}a^3v}\partial_5(a\pi) , \qquad \pi \equiv A_5 . \tag{13}$$

After integration by parts the 5D quadratic terms for the gauge bosons and the pseudoscalar π are

$$\mathcal{L}_{V} = \frac{aM_{5}}{2} \operatorname{Tr} \left\{ V_{\mu} \left[(\partial^{2} - a^{-1}\partial_{5}a\partial_{5})\eta_{\mu\nu} - \partial_{\mu}\partial_{\nu} \right] V_{\nu} \right\} ,$$

$$\mathcal{L}_{A} = \frac{aM_{5}}{2} \operatorname{Tr} \left\{ A_{\mu} \left[(\partial^{2} - a^{-1}\partial_{5}a\partial_{5} + 2v^{2}a^{2})\eta_{\mu\nu} - \partial_{\mu}\partial_{\nu} \right] A_{\nu} \right\} ,$$

$$\mathcal{L}_{\pi} = \frac{M_{5}}{2} \operatorname{Tr} \left[a(\partial_{\mu}\pi)^{2} + \frac{1}{2a^{3}v^{2}} (\partial_{\mu}\partial_{5}(a\pi))^{2} - 2v^{2}a^{3} \left(\pi - \partial_{5} \left[\frac{1}{2a^{3}v^{2}} \partial_{5}(a\pi) \right] \right)^{2} \right] .$$
(14)

There are also boundary terms

$$\mathcal{L}_{\text{bound}} = \frac{aM_5}{2} \operatorname{Tr} \left(V_{\mu} \partial_5 V_{\mu} - 2V_{\mu} \partial_{\mu} V_5 + A_{\mu} \partial_5 A_{\mu} - 2A_{\mu} \partial_{\mu} \pi \right) \Big|_{L_0}^{L_1}. \tag{15}$$

The IR-boundary terms can be canceled by imposing the following boundary conditions:

$$\partial_5 V_\mu \big|_{L_1} = V_5 \big|_{L_1} = \partial_5 A_\mu \big|_{L_1} = \pi \big|_{L_1} = 0.$$
 (16)

The UV-boundary conditions will be discussed later. Other important quantities that we will be interested are the vertices:

$$\mathcal{L}_{VA\pi} = i\sqrt{2}aM_5 \operatorname{Tr} \left[A_{\mu} [\partial_5 V_{\mu}, \pi] + \frac{1}{2} A_{\mu} [V_{\mu}, A_5] \delta(z - L_0) \right], \tag{17}$$

$$\mathcal{L}_{V\pi\pi} = \frac{iaM_5}{\sqrt{2}} \operatorname{Tr} \left(\partial_{\mu} \pi [V_{\mu}, \pi] \right) + \frac{iM_5}{2\sqrt{2}a^3 v^2} \operatorname{Tr} \left(\partial_{\mu} \partial_5 (a\pi) [V_{\mu}, \partial_5 (a\pi)] \right) , \tag{18}$$

$$\mathcal{L}_{\pi^4} = \frac{M_5}{48a^9v^6} \operatorname{Tr} \left[(\partial_5(a\pi) \partial_\mu \partial_5(a\pi))^2 - (\partial_\mu \partial_5(a\pi))^2 (\partial_5(a\pi))^2 \right]. \tag{19}$$

3.1 The current-current correlators $\Pi_{V,A}$

In QCD the generating functional of the current-current correlators is calculated by integrating out the quarks and gluons as a function of the external sources. This must be equivalent in the large- N_c limit to integrate all the colorless resonances at tree-level. The AdS/CFT correspondence tells us that this generating functional is the result of integrating out, at tree-level, the 5D gauge fields restricted to a given UV-boundary value:

$$V_{\mu}\big|_{L_0} = v_{\mu} , \qquad A_{\mu}\big|_{L_0} = a_{\mu} .$$
 (20)

The boundary fields v_{μ} and a_{μ} play the role of external sources coupled respectively to the vector and axial-vector QCD currents. At the quadratic level the generating functional is simple to calculate. We have to solve the equations of motion Eqs. (14) for the 5D gauge fields restricted to the UV-boundary condition Eq. (20), and substitute the solution back into the action. This leads to the effective lagrangian that gives the generating functional of the two-point correlators $\Pi_{V,A}$:

$$\mathcal{L}_{\text{eff}} = \frac{P_{\mu\nu}}{2} \operatorname{Tr} \left[v_{\mu} \Pi_{V}(p^{2}) v_{\nu} + a_{\mu} \Pi_{A}(p^{2}) a_{\nu} \right] . \tag{21}$$

We are working in momentum space and $P_{\mu\nu} = \eta_{\mu\nu} - p_{\mu}p_{\nu}/p^2$. For the AdS₅ space the Π_V can be calculated analytically [17, 14]:

$$\Pi_V(p^2) = -M_5 L \frac{ip}{L_0} \frac{J_0(ipL_1)Y_0(ipL_0) - J_0(ipL_0)Y_0(ipL_1)}{J_0(ipL_1)Y_1(ipL_0) - J_1(ipL_0)Y_0(ipL_1)},$$
(22)

where J_n, Y_n are Bessel functions of order n and p is the Euclidean momentum. For large momentum, $pL_1 \gg 1$, the dependence on p of the correlators is dictated by the conformal symmetry and we find

$$\Pi_V(p^2) \simeq -\frac{M_5 L}{2} p^2 \ln(p^2 L_0^2).$$
(23)

 $1/L_0$ plays the role of a UV-cutoff that can be absorbed in the bare kinetic term of v_{μ} . The coefficient of Eq. (23) must be matched to the QCD β -function of Eq. (1). We get

$$M_5 L = \frac{N_c}{12\pi^2} \equiv \tilde{N}_c \,, \tag{24}$$

that fixes the value of the 5D coupling. The next to leading terms in the large momentum expansion of Eq. (23) appear suppressed exponentially with the momentum $\sim e^{-pL_1}$, contrary to the QCD Π_V correlator of Eq. (1). This is because in our 5D model the vector V_M does not couple to $\langle \Phi \rangle^2$, and therefore it does not feel the breaking of the conformal symmetry coming from $\langle \Phi \rangle^2$. In fact the only breaking of the conformal symmetry that V_M feels arises from the IR-boundary that sharply cuts the AdS₅ space, but these effects decouple exponentially at large momentum. To reproduce the extra terms of Eq. (1), we would have to consider either higher-dimensional operators mixing V_M with $\langle \Phi \rangle^2$ or IR deviations from the AdS₅ space.

In large- N_c theories the correlators $\Pi_{V,A}$ can be rewritten as a sum over narrow resonances:

$$\Pi_V = p^2 \sum_n \frac{F_{V_n}^2}{p^2 + M_{V_n}^2}, \quad \Pi_A = p^2 \sum_n \frac{F_{A_n}^2}{p^2 + M_{A_n}^2} + F_{\pi}^2.$$
(25)

 F_{V_n} and F_{A_n} are the vector and axial-vector decay constants and the poles of $\Pi_{V,A}$ give the mass spectrum. The correlators $\Pi_{V,A}$ calculated via the AdS/CFT correspondence can also be rewritten as in Eq. (25). For the AdS₅ space the masses M_{V_n} are determined by the poles of Eq. (22):

$$J_0(M_{V_n}L_1) \simeq 0 \longrightarrow M_{V_n} \simeq \left(n - \frac{1}{4}\right) \frac{\pi}{L_1}.$$
 (26)

For the n=1 resonance, the rho meson, we have $M_{\rho} \simeq 2.4/L_1$ that we will use to determine the value of L_1

$$M_{\rho} \simeq 770 \text{ MeV} \to \frac{1}{L_1} \simeq 320 \text{ MeV}.$$
 (27)

The vector decay constants are given by the residues of the poles of Π_V/p^2 . We obtain

$$F_{V_n}^2 = \tilde{N}_c \frac{\pi M_{V_n}}{L_1} \frac{Y_0(M_{V_n} L_1)}{J_1(M_{V_n} L_1)}.$$
 (28)

Using Eqs. (24), (27) and (28) we obtain $F_{V_1} \simeq 140$ MeV to be compared with the experimental value $F_{\rho} = 153$ MeV. For the higher resonances we obtain $F_{V_{2,3}} \simeq 210, 270$ MeV.

The correlator Π_A depends on the z-dependent mass of A_{μ} and cannot be calculated analytically. Numerical analysis is therefore needed to obtain the masses and decay constants of the axial-vector mesons. Analytical formulas, however, can be obtained if we approximate the 5D mass of A_{μ} as a IR-boundary 4D mass, $M_{\rm IR}$. This is expected to be a good approximation since the scalar VEV v(z) that gives a mass to A_{μ} grows towards the IR-boundary as $v(z) \simeq (z/L_1)^3 \xi/L$ and is only relevant for values of z close to the IR-boundary. The value of $M_{\rm IR}$ is determined by

$$\int_{L_0}^{L_1} dz \, a^3(z) M_{\rm IR}^2 A_\mu \, \delta(z - L_1) = M_5 \int_{L_0}^{L_1} dz \, 2a^3(z) v^2(z) A_\mu \,. \tag{29}$$

The effect of a IR-boundary mass is simply to change the IR-boundary condition from Eq. (16) to $\left[M_5\partial_5+a^2M_{\rm IR}^2\right]A_\mu\Big|_{L_1}=0$ [18], and therefore the equation that determines the mass spectrum changes from Eq. (26) to

$$J_0(M_{A_n}L_1) \simeq -\int_{L_0}^{L_1} dz \, \frac{2a^3(z)v^2(z)}{M_{A_n}L} z J_1(M_{A_n}z) \,. \tag{30}$$

In Fig. 1 we show the value of the mass of the lowest state as a function of ξ . We compare the exact numerical value of M_{A_1} and the approximate value coming from Eq. (30). We see that the difference is below the 10%. For $\xi \simeq 4$ we find that M_{A_1} coincides with the experimental mass of the a_1 , $M_{a_1} \simeq 1230$ MeV. We then see that the experimental data favor values of ξ around 4. For this value $\xi \simeq 4$ we also find that $F_{A_1} \simeq 160$ MeV. For the second resonance we find, for $\xi = 4$, $M_{A_2} \simeq 2$ GeV and $F_{A_2} \simeq 200$ MeV. For heavier axial-vector resonances the right-hand side of Eq. (30) can be neglected and then their masses approach to the values of the vector masses Eq. (26) (and similarly for the decay constants).

In the large and small momentum limits the correlator Π_A can also be calculated analytically without the need of the above approximation. Furthermore these analytical expressions simplify

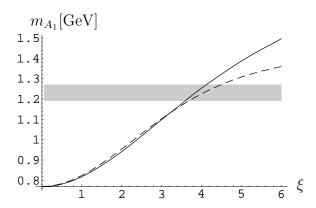


Figure 1: Mass of the first axial-vector resonance as a function of ξ . The solid line is the exact result, while the dashed line corresponds to the approximate value coming from Eq. (30). The shadow band shows the experimental value $M_{a_1} = 1230 \pm 40$ MeV.

enormously if $\xi \gg 1$. In this limit we find that the dependence of Π_A on ξ is simply dictated by the conformal symmetry. For small p, we have

$$\Pi_A(p^2) = \Pi_A(0) + p^2 \Pi'_A(0) + \mathcal{O}(p^4), \qquad (31)$$

where for $\xi \gg 1$

$$\Pi_A(0) = F_\pi^2 \simeq \frac{2^{5/3}\pi}{3^{1/6}\Gamma(\frac{1}{3})^2} \frac{\tilde{N}_c \, \xi^{2/3}}{L_1^2} ,$$
(32)

$$\Pi_A'(0) \simeq -\tilde{N}_c \left[\ln \frac{L_0}{L_1} + \ln \xi^{1/3} + \frac{4\gamma + \pi\sqrt{3} - \ln 12}{12} \right].$$
 (33)

From Eq. (32), making use of Eqs. (24) and (27), we get

$$F_{\pi} \simeq 87 \left(\frac{\xi}{4}\right)^{\frac{1}{3}} \text{ MeV},$$
 (34)

in excellent agreement with the experimental value for $\xi \simeq 4$. We have checked that the approximate value of F_{π} Eq. (34) differs from the exact value by less than 10% if $\xi \gtrsim 3$. Adding an explicit breaking of the chiral symmetry, $M_q \neq 0$, gives an extra contribution to $\Pi_A(0)$. By expanding around $M_q = 0$, we obtain $\Pi_A(0) = \Pi_A^{(0)}(0) + M_q L_1 \Pi_A^{(1)}(0) + \cdots$, where, in the limit $\xi \gg 1$, $\Pi_A^{(0)}$ is given by Eq. (32) and

$$\Pi_A^{(1)}(0) \simeq \frac{2^{8/3} 3^{2/3} \pi}{\Gamma(\frac{1}{6})^2} \frac{\tilde{N}_c \, \xi^{1/3}}{L_1^2} \left(1 - \frac{2\Gamma(\frac{1}{6})}{6^{4/3} \sqrt{\pi}} \frac{1}{\xi^{2/3}} \right) \,. \tag{35}$$

Eq. (35) gives a contribution to F_{π} proportional to the quark masses M_q .

In the large momentum limit Π_A is given in the chiral limit by

$$\Pi_A(p^2) = -p^2 \left[\frac{\tilde{N}_c}{2} \ln(p^2 L_0^2) + \frac{c_6}{p^6} + \mathcal{O}(\frac{1}{p^{12}}) \right] , \quad \text{where} \quad c_6 = -\frac{16}{5} \frac{\tilde{N}_c \, \xi^2}{L_1^6} . \quad (36)$$

As we said before, corrections to Eq. (36) are expected if the 5D metric departs in the IR from AdS. Nevertheless, these corrections cancel out in the left-right correlator $\Pi_{LR} = \Pi_V - \Pi_A$. Therefore, at large momentum we have

$$\Pi_{LR}(p^2) \simeq \frac{c_6}{p^4} + \cdots, \tag{37}$$

independently of variations in the AdS_5 metric. Eq. (36) gives

$$c_6 \simeq -1.4 \times 10^{-3} \left(\frac{\xi}{4}\right)^2 \text{GeV}^6,$$
 (38)

to be compared with the QCD value $c_6 = -4\pi\alpha_s\langle\bar{q}q\rangle^2 \simeq -1.3\times 10^{-3}~{\rm GeV^6}$ extracted from the evaluation of the condensate of Ref. [19]. We must notice, however, that Eq. (37) will be affected by higher-dimensional operators such as ${\rm Tr}[\Phi^{\dagger}L_{MN}\Phi R^{MN}]$.

3.2 Vector meson interactions

To calculate the couplings between the resonances it is convenient to perform a Kaluza-Klein (KK) decomposition of the 5D fields. This consists in expanding the fields in a tower of 4D mass-eigenstates, $V_{\mu}(x,z) = \frac{1}{\sqrt{M_5L}} \sum_{n} f_n^V(z) V_{\mu}^{(n)}(x)$, and equivalently for the other fields. To cancel the UV-boundary terms of Eq. (15) we impose

$$V_{\mu}\big|_{L_0} = A_{\mu}\big|_{L_0} = 0. \tag{39}$$

For the electromagnetic subgroup of $SU(3)_V$, however, we must consider the boundary condition $\partial_5 V_\mu|_{L_0} = 0$ in order to have a massless mode in the spectrum, the photon, whose wave-function satisfies

$$\partial_5 f_0^V = 0. (40)$$

In the limit $L_0 \to 0$ this state becomes non-normalizable since its kinetic term diverges as $\tilde{N}_c \ln (L_1/L_0)$. To keep it as a dynamical field, we can fix $1/L_0$ to a large but finite value. In the absence of UV-boundary kinetic terms, this value of $1/L_0$ defines the scale of the Landau pole [17]: ¹

$$\frac{1}{e^2(\mu)} = -\tilde{N}_c \ln(L_0 \mu) \,. \tag{41}$$

The wave-functions of the KK modes $V_{\mu}^{(n)}$ $(n \neq 0)$ are given for the AdS₅ case by [18]:

$$f_n^V(z) = \frac{z}{N_{V_n} L_1} \left[J_1(M_{V_n} z) + b(M_{V_n}) Y_1(M_{V_n} z) \right] \xrightarrow{L_0 \to 0} \frac{z}{N_{V_n} L_1} J_1(M_{V_n} z) , \tag{42}$$

where $b(M_{V_n}) = -\frac{J_1(M_{V_n}L_0)}{Y_1(M_{V_n}L_0)}$ and N_{V_n} is a constant fixed by canonically normalizing the field. The masses M_{V_n} are determined by the condition $\partial_5 f_n^V(z)\big|_{L_1} = 0$ that coincides with the poles

¹This is equivalent to add a kinetic term on the UV-boundary with coupling $1/e_0^2 = \tilde{N}_c \ln{(L_0\mu)} + 1/e^2(\mu)$ that cancels, in the limit $L_0 \to 0$, the divergence in the kinetic term of the massless mode and normalizes this state.

of Eq. (22). For the vector KK modes associated to the electromagnetic subgroup, we have $b(M_{V_n}) = -\frac{J_0(M_{V_n}L_0)}{Y_0(M_{V_n}L_0)}$. In this case the KK masses are different by factors of order e^2 from the values of Eq. (26). This is expected since the KK modes are mass-eigenstates and their masses incorporate corrections due to the mixing of the resonances in Eq. (25) with the photon. In Fig. 2 we plot the wave-function of the first two KK modes.

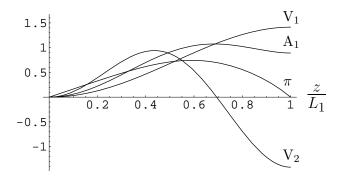


Figure 2: Wave-function of the n = 1, 2 vector resonance, the n = 1 axial-vector resonance and the PGB.

For the axial-vector A_{μ} there is no massless mode. The KK wave-functions cannot be obtained analytically and one must rely in numerical analysis. In Fig. 2 we plot the wave-function of the first KK mode, the a_1 , for the AdS₅ case. Throughout this section we will work in the chiral limit.

Finally, the pseudoscalar fields π have also a 4D massless mode corresponding to the PGB arising from chiral symmetry breaking. Their wave-functions are determined by

$$\left[\frac{1}{a} - \partial_5 \left(\frac{1}{2a^3v^2}\partial_5\right)\right] a f_0^{\pi} = 0.$$

$$\tag{43}$$

For AdS_5 we obtain

$$f_0^{\pi} = \frac{z^3}{L_1^3 N_0} \left[I_{2/3} \left(\frac{\sqrt{2}\xi}{3} \frac{z^3}{L_1^3} \right) - \frac{I_{2/3} \left(\sqrt{2}\xi/3 \right)}{K_{2/3} \left(\sqrt{2}\xi/3 \right)} K_{2/3} \left(\frac{\sqrt{2}\xi}{3} \frac{z^3}{L_1^3} \right) \right], \tag{44}$$

where the constant N_0 canonically normalizes the field (this is fixed by $\frac{1}{2a^2v^2L}f_0^{\pi}\partial_5(af_0^{\pi})|_{L_0}=1$). Here an after we will denote by π only the massless modes, the PGB. Their wave-functions are shown in Fig. 2.

The interactions between the different resonances are easily obtained from Eqs. (17)-(19) and integrating over z with the corresponding wave-functions. Here we present some phenomenologically relevant examples. The first one is the coupling of the photon to $A_{\mu}^{(n)}\pi$. Using Eqs. (17), (39) and (40), we obtain that this coupling is zero:

$$g_{A_n\gamma\pi} = 0. (45)$$

Eq. (45) is a consequence of electromagnetic gauge invariance which implies that $p^{\nu}\mathcal{M}_{\mu\nu} = 0$ where p^{ν} is the momentum of the photon and $\mathcal{M}_{\mu\nu}$ is the vertex $A_{\mu}^{(n)}\gamma_{\nu}\pi$. In the 5D model of Eq. (5), in which only dimension 4 operators are considered, we have at tree level that $\mathcal{M}_{\mu\nu}$ can only be proportional to $g_{\mu\nu}$ and then Eq. (45) follows from the condition of gauge invariance. Eq. (45) has the interesting consequence that, at the leading order in large- N_c , the branching ratio of $a_1 \to \gamma \pi$ vanishes. This coupling, however, could be induced from 5D higher-dimensional operators or quantum loop effects.

Another example is the vector coupling to two PGB. From Eq. (18) we get

$$\mathcal{L}_{V_n\pi\pi} = i \frac{g_{n\pi\pi}}{\sqrt{2}} \operatorname{Tr} \left(\partial_{\mu} \pi [V_{\mu}^{(n)}, \pi] \right), \tag{46}$$

where $g_{n\pi\pi}$ is given by

$$g_{n\pi\pi} = \frac{1}{\sqrt{M_5 L^3}} \int dz \, a f_n^V \left[(f_0^{\pi})^2 + \frac{(\partial_5 (a f_0^{\pi}))^2}{2a^4 v^2} \right] . \tag{47}$$

In Fig. 3 we show the coupling of the first three KK modes as a function of ξ for the AdS₅ case. One can see that the heavier is the KK mode (larger n), the smaller is its coupling to PGB. This can be understood as a consequence of the increase in the oscillations of the KK wave-function as n increases (see Fig. 2), that implies a smaller contribution to the integral Eq. (47) for larger n.

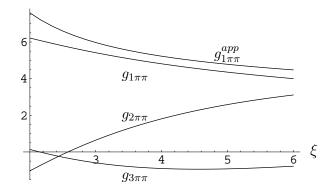


Figure 3: Coupling of the n=1,2,3 vector resonance to two PGB. We also show the approximate value for n=1 given by $g_{1\pi\pi}^{app}=M_{V_1}/(\sqrt{3}\,F_\pi)$ -see Eq. (49).

Finally, we consider the four-pion interaction. It receives contributions coming from the exchange of vector resonances, scalar resonances, and the four-interaction of Eq. (19). At the order p^2 , the chiral symmetry tells us that this coupling must be $(1/12F_{\pi}^2) \operatorname{Tr}[(\pi \overleftrightarrow{\partial_{\mu}} \pi)^2]$. This implies the following sum rule:

$$g_{\pi^4} + \sum_n \frac{g_{n\pi\pi}^2}{M_{V_n}^2} = \frac{1}{3F_\pi^2},\tag{48}$$

where g_{π^4} denotes the scalar contributions and the four-interaction. We find that for values $\xi \gtrsim 3$ the contribution g_{π^4} is small and only the vector contribution dominates. This is saturated

(at the 90% level) by the first resonance, the rho meson, leading to the following approximate relation

$$M_{\rho}^2 \simeq 3F_{\pi}^2 g_{\rho\pi\pi}^2 \,. \tag{49}$$

In Fig. 3 we plot the approximate value of $g_{1\pi\pi}$ that arises from Eq. (49), and it is shown that the difference from its exact value is only $\sim 10\%$. Eq. (49) differs by a factor 2/3 from the KSRF relation [20], $M_{\rho}^2 \simeq 2F_{\pi}^2g_{\rho\pi\pi}^2$, that is known to be experimentally very successful. The approximate relation Eq. (49) had been found previously in a specific extra dimensional model [7]. We have shown here, however, that it is a general prediction of 5D models independent of the space geometry. It only relies on the 5D gauge symmetry that forbids terms with four A_5 .

3.3 The electromagnetic form factor of the pion

The electromagnetic form factor of the pion, $\mathcal{F}_{\pi}(p)$ where p is the momentum transfer, corresponds to the coupling of the pion to the external vector field v_{μ} . In the 5D picture the pion does not couple directly to v_{μ} but only through the interchange of the vector resonances. This is because the pion wave-function is zero at the UV-boundary and therefore the pion can only couple to the UV-boundary fields through the KK states. This implies that the form factor of the pion can be written as

$$\mathcal{F}_{\pi}(p) = \sum_{n} g_{n\pi\pi} \frac{M_{V_n} F_{V_n}}{p^2 + M_{V_n}^2} \,. \tag{50}$$

The quantization of the electric charge of the pion implies $\mathcal{F}_{\pi}(0) = 1$ from which one can derive the sum rule $\sum_{n} g_{n\pi\pi} F_{V_n}/M_{V_n} = 1$. Above we saw that the coupling $g_{n\pi\pi}$ and the inverse of the mass decrease as n increases implying that this sum rule is mostly dominated by the first resonance and therefore

$$g_{\rho\pi\pi}F_{\rho} \simeq M_{\rho}$$
 (51)

For $\xi \simeq 4$, this relation is fulfilled at the 88% level. For larger values of ξ , however, Eq. (51) is not so well satisfied since the contribution of the second resonance becomes important. Eq. (51) together with Eq. (49) allows us to write a relation between the ρ and π decay constants

$$F_{\rho} \simeq \sqrt{3}F_{\pi} \,. \tag{52}$$

At large momentum the contribution of each n > 1 resonance to $\mathcal{F}_{\pi}(p)$ becomes sizable since the small value of $g_{n\pi\pi}$ for n > 1 is compensated by the large value of $M_{V_n}F_{V_n}$. Nevertheless, the total contribution coming from summing over all the modes with n > 1 approximately cancels out, implying that the rho meson dominates in Eq. (50) even at large momentum. This can be seen in Fig. 4 where we compare the exact result for $\mathcal{F}_{\pi}(p)$ to the result in which only one resonance is considered $\mathcal{F}_{\pi}(p) = M_{\rho}^2/(p^2 + M_{\rho}^2)$. The cancellation of the contribution of the heavy modes to $\mathcal{F}_{\pi}(p)$ is a consequence of the conformal symmetry. At large momentum transfer the conformal symmetry tells us that the electromagnetic form factor of a scalar hadron drops as $1/p^{(2\tau_h-2)}$ where $\tau_h = \text{Dim}[\mathcal{O}_h] - s$ being \mathcal{O}_h the local operator that creates the hadron from the vacuum and s the spin of the operator [4]. For the case of the pion we have that $\tau_h = 2$ (where \mathcal{O}_h is the axial-vector current operator) and then $\mathcal{F}_{\pi}(p)$ must drop as $1/p^2$. This large momentum behaviour coincides with that of the rho contribution to $\mathcal{F}_{\pi}(p)$.

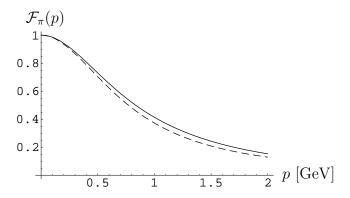


Figure 4: Electromagnetic pion form factor as a function of the transfer momentum p for $\xi = 4$. The solid line is the exact result, while the dashed line is obtained by considering only the rho meson (VMD).

The hypothesis that the electromagnetic form factor of the pion is dominated by the rho meson, that goes under name of VMD, was proposed long ago. It did not have any theoretical motivation, but it led to a good agreement with experiments tough. We have seen, however, that VMD in $\mathcal{F}_{\pi}(p)$ appears as an unavoidable consequence of this 5D model for $\xi \sim 4$ (see also Ref. [7]).

3.4 The effective lagrangian for the ρ meson

We have seen that the rho meson gives the largest contribution to the pion couplings. Therefore in order to obtain the chiral lagrangian for the PGB, it is convenient to write first the effective lagrangian for the rho meson.

In order to make contact with the literature [21], we will write the effective lagrangian not in the mass-eigenstate basis but in the basis defined by v_{μ} as in Eq. (20) and the rho field V_{μ} transforming under the SU(3)_V symmetry as $V_{\mu} \to hV_{\mu}h^{\dagger} + i/g\,h\partial_{\mu}h^{\dagger}$. From now on we will follow the notation and definitions of Ref. [21]. The effective lagrangian for V_{μ} invariant under the chiral symmetry can be written as

$$\mathcal{L}_{V} = -\frac{1}{4} \operatorname{Tr}[V_{\mu\nu}V^{\mu\nu}] + \frac{1}{2} M_{\rho}^{2} \operatorname{Tr}\left[V_{\mu} - \frac{i}{g}\Gamma_{\mu}\right]^{2} - \frac{\widetilde{F}_{\rho}}{2\sqrt{2}M_{\rho}} \operatorname{Tr}[V_{\mu\nu}f_{+}^{\mu\nu}] + \cdots,$$
 (53)

where

$$\Gamma_{\mu} = \frac{1}{2} \left\{ u^{\dagger} (\partial_{\mu} - iR_{\mu}) u + u (\partial_{\mu} - iL_{\mu}) u^{\dagger} \right\} , \qquad f_{+}^{\mu\nu} = u F_{L}^{\mu\nu} u^{\dagger} + u^{\dagger} F_{R}^{\mu\nu} u , \qquad (54)$$

and $u^2 = U$ being U a parametrization of the PGB:

$$U = e^{i\sqrt{2}\pi/F_{\pi}}. (55)$$

The lagrangian Eq. (53) does not contain all possible chiral terms of $\mathcal{O}(p^4)$. We have neglected couplings between π and V_{μ} involving more than one derivative since these couplings do not

arise from a 5D lagrangian. (We have also neglected trilinear couplings between vectors since they do not play any role in our analysis).

Matching the above lagrangian with the 5D AdS theory we obtain

$$\frac{1}{g} = 2\sqrt{2}g_{\rho\pi\pi}\frac{F_{\pi}^{2}}{M_{\rho}^{2}},$$

$$\widetilde{F}_{\rho} = F_{\rho} - \frac{M_{\rho}}{\sqrt{2}g}.$$
(56)

Notice that our 5D model predicts a nonzero value for \widetilde{F}_{ρ} (we find $\widetilde{F}_{\rho} \simeq 40$ MeV for $\xi = 4$) differently from Ref. [21] or models where the rho is considered a Yang-Mills field [22] in which one has $\widetilde{F}_{\rho} = 0$.

4 The chiral lagrangian for the PGB

By integrating all the heavy resonances we can obtain the effective lagrangian for the PGB. This lagrangian is fixed by the chiral symmetry up to some unknown coefficients. In these section we will give the prediction of the AdS_5 model for these coefficients.

The chiral lagrangian for the PGB π can be written as a function of U defined in Eq. (55). Up to $\mathcal{O}(p^2)$, this is given by

$$\mathcal{L}_2 = \frac{F_\pi^2}{4} \operatorname{Tr} \left[D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U \right], \tag{57}$$

where

$$D_{\mu}U = \partial_{\mu}U - iR_{\mu}U + iUL_{\mu}, \qquad (58)$$

and

$$\chi = 2B_0 (M_q + ip) , \qquad M_q = \text{Diag}(m_u, m_d, m_s) .$$
 (59)

The constant B_0 is related to the quark condensate by $\langle \bar{q}q \rangle = -B_0 F_{\pi}^2$. In the AdS₅ model F_{π} is given by Eq. (32), while matching Eq. (57) in the unitary gauge (U=1) to Eq. (10) we get

$$F_{\pi}^{2}B_{0} = \frac{2\tilde{N}_{c}\,\xi}{L_{1}^{3}}\,,\tag{60}$$

that determines the PGB masses:

$$(m_{\pi}^2)_{ab} = 2B_0 \operatorname{Tr} [M_q T_a T_b]$$
 (61)

From the pion mass $m_{\pi^0} \simeq 135$ MeV, we obtain $m_u + m_d \simeq 20.5$ MeV for $\xi = 4$.

At the $\mathcal{O}(p^4)$ the chiral lagrangian has extra terms given by [23]

$$\mathcal{L}_{4} = L_{1} \operatorname{Tr}^{2} \left[D_{\mu} U^{\dagger} D^{\mu} U \right] + L_{2} \operatorname{Tr} \left[D_{\mu} U^{\dagger} D_{\nu} U \right] \operatorname{Tr} \left[D^{\mu} U^{\dagger} D^{\nu} U \right] + L_{3} \operatorname{Tr} \left[D_{\mu} U^{\dagger} D^{\mu} U D_{\nu} U^{\dagger} D^{\nu} U \right]
+ L_{4} \operatorname{Tr} \left[D_{\mu} U^{\dagger} D^{\mu} U \right] \operatorname{Tr} \left[U^{\dagger} \chi + \chi^{\dagger} U \right] + L_{5} \operatorname{Tr} \left[D_{\mu} U^{\dagger} D^{\mu} U \left(U^{\dagger} \chi + \chi^{\dagger} U \right) \right]
+ L_{6} \operatorname{Tr}^{2} \left[U^{\dagger} \chi + \chi^{\dagger} U \right] + L_{7} \operatorname{Tr}^{2} \left[U^{\dagger} \chi - \chi^{\dagger} U \right] + L_{8} \operatorname{Tr} \left[\chi^{\dagger} U \chi^{\dagger} U + U^{\dagger} \chi U^{\dagger} \chi \right]
- i L_{9} \operatorname{Tr} \left[F_{R}^{\mu\nu} D_{\mu} U D_{\nu} U^{\dagger} + F_{L}^{\mu\nu} D_{\mu} U^{\dagger} D_{\nu} U \right] + L_{10} \operatorname{Tr} \left[U^{\dagger} F_{R}^{\mu\nu} U F_{L\mu\nu} \right].$$
(62)

The coefficients $L_{1,2,3}$ are responsible for four-pion interactions at $\mathcal{O}(p^4)$, while L_9 gives a contribution to the electromagnetic form factor of the pion at $\mathcal{O}(p^2)$. From the discussion of the previous section we know that the dominant contribution to these processes arises from the rho meson exchange. Therefore the main contribution to $L_{1,2,3,9}$ will arise by integrating out this particle. Using the effective lagrangian Eq. (53) with Eqs. (56), we obtain ²

$$L_1 = \frac{g_{\rho\pi\pi}^2 F_{\pi}^4}{8M_{\varrho}^4}, \tag{63}$$

$$L_2 = 2L_1, L_3 = -6L_1, (64)$$

$$L_9 = \frac{g_{\rho\pi\pi} F_{\rho} F_{\pi}^2}{2M_{\rho}^3} \,. \tag{65}$$

Using Eqs. (49) and (51), we get

$$L_1 \simeq \frac{F_\pi^2}{24M_\rho^2} \,, \qquad L_9 \simeq \frac{F_\pi^2}{2M_\rho^2} \,.$$
 (66)

The coefficients $L_{4,6}$ are zero at the tree-level (leading order in the large- N_c expansion), while the calculation of $L_{7,8}$ will be left for the future. L_7 involves the U(1) anomaly and L_8 only receives contributions from the scalar sector. L_5 and L_{10} can be calculated from the correlators $\Pi_{V,A}$:

$$L_5 = \frac{L_1}{8B_0} \Pi_A^{(1)}(0), \quad \text{(only if } L_4 = 0),$$
 (67)

$$L_{10} = \frac{1}{4} \left[\Pi'_A(0) - \Pi'_V(0) \right], \tag{68}$$

where L_1 in Eq. (67) is the one defined in Eq. (27). From Eqs. (22), (32)-(35) and (60) we obtain for $\xi \gg 1$

$$L_5 \simeq \tilde{N}_c \frac{2\pi^3}{\sqrt{3}\Gamma(\frac{1}{3})^6} \left[1 - \frac{2\Gamma(\frac{1}{6})}{6^{4/3}\sqrt{\pi}} \frac{1}{\xi^{2/3}} \right] \simeq 2.5 \cdot 10^{-3} \left[1 - 0.23 \left(\frac{4}{\xi} \right)^{\frac{2}{3}} \right] , \tag{69}$$

$$L_{10} \simeq -\frac{\tilde{N}_c}{4} \left[\ln \xi^{1/3} + \frac{4\gamma + \pi\sqrt{3} - \ln 12}{12} \right] \simeq -5.7 \cdot 10^{-3} \left[\ln \left(\frac{\xi}{4} \right)^{\frac{1}{3}} + 1 \right]. \tag{70}$$

²These coefficients are induced after performing the redefinition $V_{\mu} \to V_{\mu} + i\Gamma_{\mu}/g$ in Eq. (53). After this redefinition the rho meson couples to the pion only at $\mathcal{O}(p^3)$ and then it does not induce a contribution to Eq. (62) when it is integrated out.

In Table 1 we compare the experimental values of L_i with the predictions of our AdS₅ model for the value $\xi = 4$. We give the exact values of our predictions although we find that the predictions in the limit $\xi \gg 1$ differ by less than a 10% from the exact results. Comparing the predictions with the experimental values we find that the discrepancy is always below the 30%.

	Experiment	AdS_5
L_1	0.4 ± 0.3	0.4
L_2	1.4 ± 0.3	0.9
L_3	-3.5 ± 1.1	-2.6
L_4	-0.3 ± 0.5	0.0
L_5	1.4 ± 0.5	1.7
L_6	-0.2 ± 0.3	0.0
L_9	6.9 ± 0.7	5.4
L_{10}	-5.5 ± 0.7	-5.5

Table 1: Experimental values of the L_i at the scale M_{ρ} in units of 10^{-3} [25] and the predictions of the AdS_5 model for the value $\xi = 4$.

Finally, we also calculate the coefficient of the operator $\text{Tr}[Q_R U Q_L U^{\dagger}]$ responsible for the electromagnetic pion mass difference $(Q_{L,R}$ are the left- and right-handed charges) [24]. This coefficient is given by $e^2 C = (m_{\pi^+}^2 - m_{\pi^0}^2) F_{\pi}^2/2$ where

$$m_{\pi^+} - m_{\pi^0} \simeq \frac{3\alpha}{8\pi m_{\pi} F_{\pi}^2} \int_0^{\infty} dp^2 (\Pi_A - \Pi_V) \,.$$
 (71)

Taking Π_V from Eq. (22) and calculating Π_A numerically in the chiral limit for $\xi = 4$ (5) we find $\Delta m_{\pi} \simeq 3.6$ (4) MeV to be compared with the experimental value $\Delta m_{\pi} \simeq 4.6$ MeV.

The coefficients L_i and C have been previously calculated using different approximations. For example, in Refs. [24, 26] these coefficients were calculated from an effective theory of resonances, showing a good agreement with the experimental data. It would be interesting to study the relation between the approach presented here with those of Refs. [24, 26].

5 Conclusions

We have presented a 5D model that describes some of the properties of QCD related to chiral symmetry breaking. Alike large- N_c QCD, this model is defined by a set of infinite weakly coupled resonances. The model depends only on one parameter, ξ , related to the quark condensate (apart from the other 3 parameters of the model that are fixed by the 3 parameters that define QCD: the mass gap $\Lambda_{\rm QCD}$, M_q , and N_c). We have obtained predictions for the masses and decay constants of the vector, axial-vector and PGB mesons. These predictions are in good agreement with the experimental data. A summary of some of the results is given in Table 1 and Fig. 5 that shows that, within a 30%, they agree with the data.

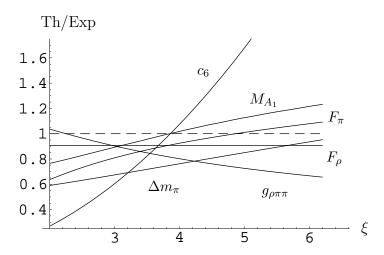


Figure 5: Predictions of the model for some physical quantities as a function of ξ divided by their experimental value. We have taken $M_q = 0$.

The 5D gauge invariance of the model leads to interesting sum rules among the couplings and masses of the resonances from which we obtain $M_{\rho}^2 \simeq 3 g_{\rho\pi\pi}^2 F_{\pi}^2$, $F_{\rho} \simeq \sqrt{3} F_{\pi}$, and the vanishing of the BR of a_1 into $\pi\gamma$ at the tree-level. Another prediction of the model is the realization of VMD in the electromagnetic form factor of the pion.

Since the results presented here depend on the AdS₅ metric Eq. (3), one can wonder whether the results are robust under possible deviations from AdS. For example, if the backreaction on the metric due to $\langle \Phi \rangle^2$ or other possible condensates are included in the model, we expect the warp factor a(z) to depart from AdS in the IR. Nevertheless if we want the theory to be almost conformal in the UV, the warp factor for $z \ll L_1$ (where $1/L_1$ gives the mass gap) must behave as

$$a(z) \simeq \frac{L}{z} \left[1 + \sum_{i} c_i \left(\frac{z}{L_1} \right)^{d_i} \right],$$
 (72)

where c_i are numerical constants related to the singlet condensates $\langle \mathcal{O}_i \rangle$ and $d_i = \text{Dim}[\mathcal{O}_i]$. In QCD $d_i \geq 4$. Eq. (72) implies that only for values of z quite close to L_1 the metric will deviate from AdS. Therefore, unless the coefficients c_i are very large, we do not expect large deviations from our results. The c_i , however, are restricted by the curvature of the space \mathcal{R} . We have checked that for $\mathcal{R} \sim 1/L_1^2$, our results are not substantially modified by deformations of the AdS metric in the IR. As an example, we have compared some of our results to those with the metric of Ref. [2], 3 $a(z) = \frac{\pi L}{2L_1 \sin[\pi z/2L_1]}$, and we have found that the differences are smaller than 10%. We can then conclude that more realistic string constructions of QCD, such as those of Refs. [3], must lead to quantitatively similar results.

³This is the induced metric on the D7-brane on which the gauge bosons propagate in Ref. [2].

Note Added: While writing this paper, it appeared Ref. [27] that proposes the same 5D model to study the properties of the QCD hadrons.

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Appendix. Chiral symmetry breaking induced by an operator of dimension d

In this appendix we give the expression for Π_A in the different limits studied in the text for the case in which the breaking of the chiral symmetry arises from a VEV of a scalar Φ with an arbitrary 5D mass M_{Φ} . This corresponds in the CFT to turning on an operator of dimension $d = \sqrt{4 + M_{\Phi}^2 L^2} + 2$.

For small momentum we have $\Pi_A(p) = \Pi_A^{(0)}(0) + M_q L_1 \Pi_A^{(1)}(0) + p^2 \Pi_A'(0) + \cdots$ where in the limit $\xi \gg 1$:

$$\Pi_A^{(0)}(0) \simeq \frac{2^{(1-1/d)} d^{(1-2/d)} \pi}{\sin(\pi/d) \Gamma(\frac{1}{d})^2} \frac{\tilde{N}_c \xi^{2/d}}{L_1^2}, \tag{73}$$

$$\Pi_A^{(1)}(0) \simeq \frac{2^{(1-1/d)}\Gamma(\frac{2+d}{2d})\Gamma(\frac{3}{d})}{d^{(1-2/d)}\Gamma(\frac{4+d}{2d})} \frac{\tilde{N}_c \, \xi^{1-2/d}}{L_1^2} \left(1 - \frac{d^{2/d}\Gamma(\frac{4+d}{2d})\Gamma(\frac{4}{d})}{2^{1/d}\Gamma(\frac{6+d}{2d})\Gamma(\frac{1}{d})} \frac{1}{\xi^{2/d}} \right) , \tag{74}$$

$$\Pi_A'(0) \simeq -\tilde{N}_c \ln \frac{L_0}{L_1} - \tilde{N}_c \left[\ln \xi^{1/d} + \frac{\gamma + \psi \left(\frac{2+d}{2d} \right) - \psi \left(\frac{2}{d} \right) - \psi \left(\frac{1}{d} \right) - \ln \frac{d^2}{2}}{2d} \right], \quad (75)$$

where $\psi(x) = \Gamma'(x)/\Gamma(x)$.

In the large momentum limit we have

$$\Pi_A(p^2) = -p^2 \left[\frac{\tilde{N}_c}{2} \ln(p^2 L_0^2) + \frac{c_{2d}}{p^{2d}} + \cdots \right] , \qquad (76)$$

where

$$c_{2d} = -\frac{d\sqrt{\pi}}{2(d-1)} \frac{\Gamma(d)^3}{\Gamma(d+\frac{1}{2})} \frac{\tilde{N}_c \, \xi^2}{L_1^{2d}} \,. \tag{77}$$

From the above expressions we can derive L_5 and L_{10} :

$$L_5 \simeq \tilde{N}_c \frac{2^{(-2-2/d)} \pi \Gamma(\frac{2+d}{2d}) \Gamma(\frac{3}{d})}{\sin(\pi/d) \Gamma(\frac{4+d}{2d}) \Gamma(\frac{1}{d})^2} \left(1 - \frac{d^{2/d} \Gamma(\frac{4+d}{2d}) \Gamma(\frac{4}{d})}{2^{1/d} \Gamma(\frac{6+d}{2d}) \Gamma(\frac{1}{d})} \frac{1}{\xi^{2/d}} \right) , \tag{78}$$

$$L_{10} \simeq -\frac{\tilde{N}_c}{4} \left[\ln \xi^{1/d} + \frac{\gamma + \psi\left(\frac{2+d}{2d}\right) - \psi\left(\frac{2}{d}\right) - \psi\left(\frac{1}{d}\right) - \ln\frac{d^2}{2}}{2d} \right]. \tag{79}$$

The parameter L_{10} allows to calculate the Peskin-Takeuchi S parameter [15] of Technicolor-like theories where $F_{\pi} = 246$ GeV triggers the breaking of the electroweak symmetry. This is given by $S = -16\pi L_{10}$. Electroweak precision tests tell us that $S \lesssim 0.3$, a constraint difficult to be satisfied in the present models [11, 12]. From Eq. (79) we can derive the dependence of S on d. For a fixed value of F_{π} we find that the dependence on d is very weak, and S changes only a few per cent when varying d. This implies that S in these type of models is always sizable.

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